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CROSS-SECTION MEASUREMENT FOR ^{40}CA BY APPLYING AGS EQUATION IN STRIPPING REACTION

Anjana Acharya*, Rajib Lochan Nayak
* Department of Physics, Veer Surendra Sai University of Technology Burla; Odisha-768018

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ABSTRACT

By applying AGS equation the cross section values for ^{40}Ca are plotted .Here 15 point Gaussian quadrature methods are applied to convert the multidimensional equation into one dimensional form.The singularities are overcome by Sasakawa and Kowalski.

KEYWORDS: Special points and weights, Gaussian quadrature method, stripping reactions, singularities
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INTRODUCTION

We know that the cross-section is an important parameter associated with a particular event i.e.elastic collision,a specific chemical reaction,specific nuclear reaction etc involving a certain combination of beam and target material.Here we consider a reaction of the type

A+a(b+x)=B(b+x)+b

The absolute cross section values for the stripping reactions (t,p),(t,d) types are calculated by R.N.Grover et al by using a simple wave function to describe the motion of the nucleons in the triton.In both cases they obtained the reseaonable agreement with the experimental data [1].Yu.A.Berezhnoy and V.Yu.Korde found the differential and integrated cross sections for the stripping reactions on the basis of diffraction theory with allowance for nuclear surface diffuseness [2].Total reaction cross sections for light projectile nucleons (H-2,H-3,He-4) interactions with nuclei are calculated using GEANT4 models in the region of coulomb barrier at low energies by V.Uzhineus [3].

In our paper the quantum analysis of scattering theory at low value of angular momentum is related with the AGS version of Faddeev approach. The cross sections are found out by converting the one dimensional coupled AGS equation in terms of form factors.

FORMULATION

For the system of three particles, AGS equation is written as

$$U_{\alpha\beta} = \left(1-\delta_{\alpha\beta}\right)(z-H_0) + \sum_{\gamma=1}^{3}\left(1-\delta_{\alpha\gamma}\right)T_\gamma(z)G_0(z)U_{\gamma\beta}(z) \qquad (1)$$

Where $T_\gamma(z)$ is the two-body transition operator in three-body space defined by the Lippmann-Schweinger equation

$$T_\gamma(z) = V_\gamma + V_\gamma G_0(z)T_\gamma(z) \qquad (2)$$

Also

$$G_\gamma(z) = (z-H_\gamma)^{-1} = (z-H_0-V_\gamma)^{-1} \qquad (3)$$

The matrix element of the AGS operator $U_{\alpha\beta}(z)$ between the asymptotic states viz.

$< \vec{Q}_\alpha d_\alpha \sum_{L_\alpha S_\alpha} \phi^{n_\alpha}_{(L_\alpha S_\alpha)J_\alpha M_\alpha} | U_{\alpha\beta}(z) | \vec{Q}_\beta d_\beta \sum_{L_\beta S_\beta} \phi^{n_\beta}_{(L_\beta S_\beta)J_\beta M_\beta} >$ can be easily related to the cross section of the process $\beta \to \alpha$ as[4]

$$\frac{d\sigma_{\beta\to\alpha}}{d\omega} = \frac{\hbar^2}{8\pi^2\mu_\beta Q_\beta} 2\pi^4 | < \vec{Q}_\alpha d_\alpha \sum_{L_\alpha S_\alpha} \phi^{n_\alpha}_{(L_\alpha S_\alpha)J_\alpha M_\alpha} |U_{\alpha\beta}(z)| \vec{Q}_\beta d_\beta \sum_{L_\beta S_\beta} \phi^{n_\beta}_{(L_\beta S_\beta)J_\beta M_\beta} > |^2_{av} \quad (4)$$

Where Q_α and Q_β are the on-shell momenta of the initial and final particles $Q_\beta^2 = (E + \varepsilon^{n_\beta}_{B_\beta})$ and $Q_\alpha^2 = (E + \varepsilon^{n_\alpha}_{B_\alpha})$. The suffix 'av' means that the quantity is to be averaged over the initial and final states.

By choosing an angular momentum basis and applying the orthonormality condition,the AGS equation can be reduced to a one dimensional form as

$$T_{\alpha\beta}(q_\alpha q'_\beta \beta_\alpha \beta_\beta : J) = \sum_{L_\alpha S_\alpha} \sum_{L_\beta S_\beta} \iint \frac{g^{n_\alpha}_{(L_\alpha S_\alpha)J_\alpha}(p_\alpha) p_\alpha^2 dp_\alpha}{(z - p_\alpha^2 - q_\alpha^2)} < p_\alpha q_\alpha (L_\alpha S_\alpha)\beta_\alpha : J | U_{\alpha\beta}(z) | p'_\beta q'_\beta (L_\beta S_\beta)\beta_\beta : J > \frac{g^{n_\alpha}_{(L_\beta S_\beta)J_\beta}(p'_\beta) {p'_\beta}^2 dp'_\beta}{(z - {p'_\beta}^2 - {q'_\beta}^2)} \quad (5)$$

Singularities

The coupled integral equation contains two types of singularities

(i) Pole singularity
(ii) Logarithmic singularity

The propagator of the coupled integral equation contains the 'pole singularity' at the pole $u_k = Q_k = (z + \varepsilon_{Bk}^{nk})^{1/2}$ which can be omitted by applying Sasakawa's method

Elimination of the singularities

The kernel of the coupled integral equation contains the logarithmic singularity and overcame by applying Doleschall's method which contains Legendre polynomial

The two-body t-matrix in separable form is written as

$$t_k(z - u_k^2) \approx \sum_{n_k} V_k |\phi^{n_k}_k > T^{n_k}_k (z - u_k^2) < \phi^{n_k}_k | V_k \quad (6)$$

where

$$T^{n_k}_k(z - u_k^2) \approx \frac{1}{z - u_k^2 + \varepsilon^{n_k}_{B_k}} \quad (7)$$

Equation (29) will be

$$t_k(z) \equiv \sum_{n_k} V_k |\emptyset^{n_k}_k > T^{n_k}_k(z) < \emptyset^{n_k}_k | V_k \quad (8)$$

$$t_k(z) = v_k |\emptyset^{n_k}_k > T^{n_k}_k(z) < \emptyset^{n_k}_k | v_k \quad (9)$$

Fuda observed that Lovelace approximation for $t_k(z)$ though confirms to the pole behaviour ,does not satisfy Lippman Schweinger equation and hence singularity because of the continuum contribution.

The normalization constant in containing the form factor is

$$N_k^{-2} = \int \frac{\{g^{n_k}_{(L_k S_k)J_k}(r_k)\}^2 r_k^2 dr_k}{(r_k^2 + \varepsilon^{n_k}_{B_k})^2} \quad (10)$$

Quantum theory of scattering

The partial wave equation of Schrodinger equation[5]

$$\frac{d^2 U_l(r)}{dr^2} + \left[k^2 - U(r) - \frac{l(l+1)}{r^2}\right] U_l(r) = 0 \quad (11)$$

$$\Psi_{lm}(r) \equiv \frac{U_l(r)}{r} Y_{lm}(\theta, \emptyset) \quad (12)$$

$$k^2 = \left(\frac{2mE}{\hbar^2}\right)$$

$$U(r) = \left(\frac{2m}{\hbar^2}\right) V(r)$$

$$\sigma_{tot} = \int_0^\infty \frac{4\pi}{k^2} sin^2 \delta_l \, d\Omega \quad (13)$$

$$\left\{\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} + \frac{2m}{\hbar^2}\left(s_{nlj} - V(r) - V_{ls}(r)\right)\right\} R_{nlj}(r) = 0 \quad (14)$$

Centrifugal potential:

$$\frac{\hbar^2 l(l+1)}{2mr^2}$$

Wood-Saxon Potential:

$$\frac{-V_0}{1+\exp(\frac{r-R}{a})}$$

The barrier height of the potentials [centrifugal + Wood-Saxon] **α** $\frac{l(l+1)}{r^2}$

$$r \sim r_0 A^{1/3}$$

The integration in equation (13) can be divided into three intervals

(i) From 0 to $\sqrt{E}$

(ii) From $\sqrt{E}$ to $\sqrt{E+2\varepsilon_B}$

(iii) From $\sqrt{E+2\varepsilon_B}$ to ∞

$\delta = -\frac{\pi}{2} k \lambda_x \frac{v^2\,(k)}{(1+Re\ \ I)}$ (15)

$\text{Re I} = \lambda x \int P\ \frac{dQ\, Q^2 v^2\,(k)}{Q^2 - k\hat{}2}$ (16)

TABULATION

P-Points	P Weights	G	g_c	Tan(δ)	PCot(δ)
0.11	0.391	1.129	1.129	-0.4326734	-0.25423333
0.555	1.924	1.504	1.504	-0.4519351	-1.2280525
1.24	2.495	0.364	0.364	-0.756442	-1.63925314
2.02	2.495	0.104	0.104	-0.9147091	-2.20835234
2.71	1.924	0.046	0.046	-0.6169421	-4.39263261
3.115	0.391	0.029	0.029	-0.2384053	-13.065983
3.27	0.011	0.027	0.027	-0.3553907	-9.20114018
3.285	0.012	0.027	0.027	-0.3569963	-9.20177692
3.301	0.012	0.025	0.025	-0.3075939	-10.7316812
3.421	0.161	0.02	0.02	-0.2029204	-16.8588281
3.979	0.394	0.006	0.006	-0.0213429	-186.43213
5.337	0.686	0.002	0.002	-0.0031842	-1676.0681
8.683	1.117	0	0	0	
19.513	1.935	0	0	0	
97.885	4.611	0	0	0	
For Sc					

For Ca					
P Points	P Weights	g	g_c	λ	Real I
0.11	0.391	1.129	1.129	93	93.628084
0.555	1.924	1.504	1.504	93	412.93811
1.243	2.495	0.364	0.364	93	30.86577
2.022	2.495	0.104	0.104	93	2.5134468
2.712	1.924	0.046	0.046	93	0.3789349
3.712	0.391	0.029	0.029	93	0.0305948
3.155	0.011	0.027	0.027	93	0.0007462
3.27	0.01	0.027	0.027	93	0.0006784
3.285	0.011	0.025	0.025	93	0.0006397
3.301	0.161	0.02	0.02	93	0.0059926

3.421	0.394	0.006	0.006	93	0.0013198
3.979	0.686	0.001	0.002	93	6.382E-05
5.537	1.117	0	0	93	0
8.683	1.935	0	0	93	0
19.513	4.611	0	0	93	0

FIGURES

Fig 1 Cross-section versus momentum for ^{41}SC

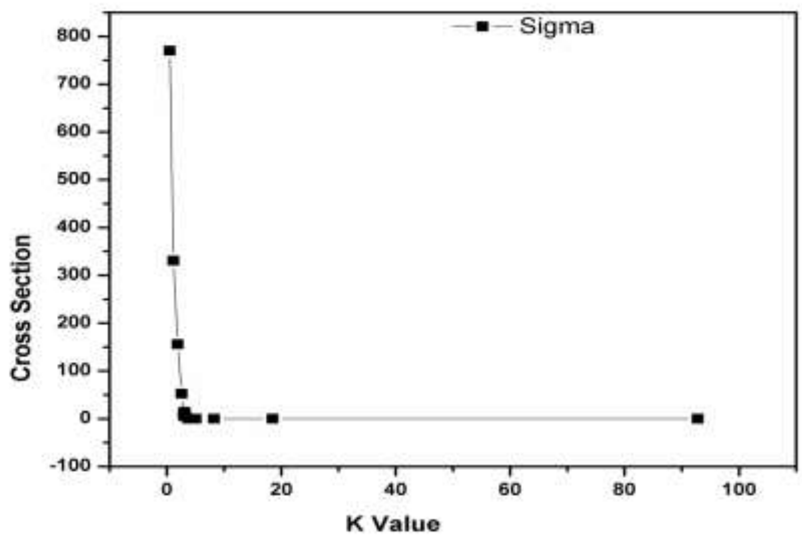

Fig 2 Cross-section versus momentum for ^{41}SC

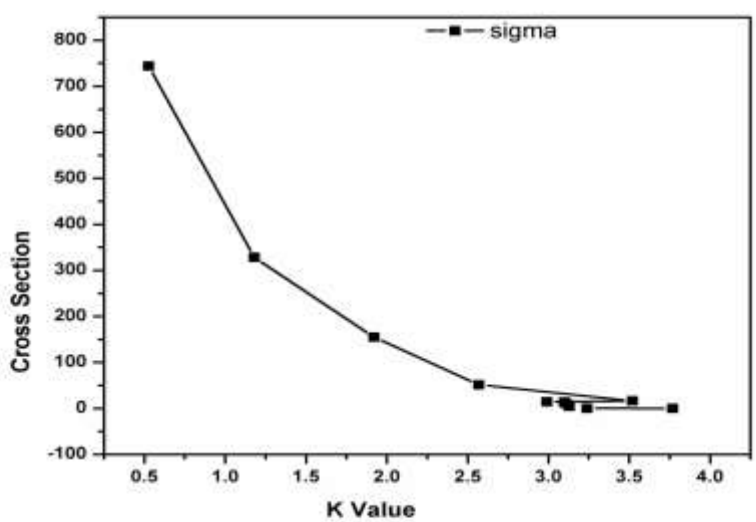

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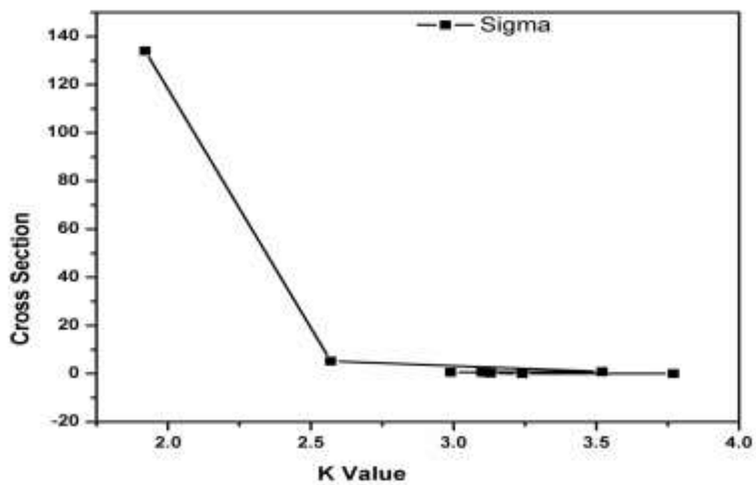
Fig 3 Cross-section versus momentum for ^{41}Ca



RESULT AND DISCUSSIONS

Within a specific range of the momentum ,the cross section is found to be approaching zero values.The minimum value is in the range 0f -20 to -100 for different types of reactions and the maximum values within the positive values of 140 to 800 as shown in the figure.The -ve values of cross section mainly occurs due to the phase shift.When we include the coulomb interaction ,the phase shift depends on the range of the interacting force. For the higher energy value, the cross section may increase depending on the type of separation energy.

CONCLUSIONS

Calcium element is taken to be studied for its stability at low partial angular momentum value.The other physical properties comparable to the other elements show some special qualities.The different isotopes of calcium near and far from the stability line give sufficient information about the nucleosynthesis.Our main aim is to further expand the area into different excited levels.

REFERENCES

[1] R.N.Glover,A.D.W. Jones and J.R.Rook "Absolute cross-sections for stripping reactions"Nuclear physics A 81 2 June 1966 289-295
[2] Yu.A.Berezhnoy and V.Yu.Korde "Deuteron stripping reaction at intermediate energy " Nuclear Physics A 556 3 17 May 1993 453-466
[3] V.Uzhinsky "Calculation of light nuclear reaction cross section in GEANT4" arXiv:1209.4455/arXiv:1209.4455v1 20 sept 2012
[4] Anjana Acharya Indian Journal of Physics 81(8) 803-817 (2007)
[5] Quantum theory of scattering by Rodberg and Thaler